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**COMMENT**

# Comment on 'The Stefan–Boltzmann constant in $n$-dimensional space'

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**Abstract.** The expression for the Stefan–Boltzmann constant in $n$-dimensions as obtained by Landsberg and De Vos is modified by the appropriate spin-degeneracy factor of the photon.

Today it is well recognized that the concept of 'dimensions' plays an important role in the theory of distribution functions, phase transitions, fractal growth, field-theoretic renormalization, superstring quantization, etc. In this context, a beautiful paper by Landsberg and De Vos [1] on the Stefan–Boltzmann constant $\sigma_n$ in $n$-dimensional space becomes particularly relevant. By a judicious combination of hyperspace geometry and Bose–Einstein statistics these authors derived

$$\sigma_n = r_n \frac{\pi^{(n-1)/2}\Gamma_{(n+1)}\zeta_{(n+1)}k^{n+1}}{\Gamma[(n+1)/2]h^n c^{n-1}} \tag{1}$$

where $r_n$ is the spin-degeneracy factor of the photon and the other symbols have their usual meaning. Assuming radiation to have always two independent states of polarization irrespective of the dimension $n$, Landsberg and De Vos (superscript LD) chose

$$r_n^{\mathrm{LD}} = 2. \tag{2}$$

The aim of the present comment is to modify expression (2) for $r_n$ by the following arguments where the polarization states of the photon will be assumed to be dictated by the dimension $n$ itself. In $n = 1$ spatial dimension there is no concept of spin, i.e. the particle would obey the massless scalar wave equation implying that $r_1 = 1$. However, in $n \geqslant 2$ dimensions the photon is described genuinely by the massless vector wave equation. A light wave travelling along a given coordinate axis can have its transverse electric vector pointing along any of the remaining $n - 1$ Cartesian axes which implies $r_n = n - 1$. Hence we suggest that in equation (1) we should use

$$r_n = \begin{cases} 1 & \text{if } n = 1 \\ n - 1 & \text{if } n \geqslant 2. \end{cases} \tag{3}$$

The prescriptions (2) and (3) coincide if $n = 3$, i.e. if the space is three-dimensional. Although the $\sigma_n$ values for $n \neq 3$ will be altered by our recipe (3), the curves of the normalized Planck spectrum given in [1] will remain unaffected.

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Before ending, the following important question must be answered: 'What is the structure of the underlying Maxwell fields in $n$-spatial dimensions?' For simplicity, we consider a region free of charges/currents and work in the radiation gauge so that the scalar potential can be made identically zero. The relevant wave equation is the massless vector one, viz

$$[\boldsymbol{\nabla}^{(n)^2} - \partial^2/c^2\partial t^2]A_i = 0 \qquad i = 1, 2, \ldots, n \tag{4a}$$

subject to the subsidiary condition

$$\boldsymbol{\nabla}^{(n)} \cdot \boldsymbol{A} = \sum_{i=1}^{n} \partial A_i/\partial x_i = 0. \tag{4b}$$

Here $\boldsymbol{\nabla}^{(n)}$ is the $n$-dimensional gradient operator, $\boldsymbol{\nabla}^{(n)^2}$ the corresponding Laplacian, $t$ the time, and $A_i$ the $i$th Cartesian component of the vector potential. Equation (4$a$) admits monochromatic plane wave solutions characterized by polarization $\boldsymbol{\epsilon}$, propagation vector $\boldsymbol{K}$, angular frequency $w = cK$ and phase $\theta$ in the form

$$\boldsymbol{A} = \boldsymbol{\epsilon} \sin(\boldsymbol{K} \cdot \boldsymbol{x} - wt + \theta) \tag{4c}$$

subject to the subsidiary condition $\boldsymbol{K} \cdot \boldsymbol{\epsilon} = 0$. The associated electric field $\boldsymbol{E}$ now becomes a vector with $n$ components given by

$$\boldsymbol{E} = -\partial \boldsymbol{A}/\partial t = w\boldsymbol{\epsilon} \cos(\boldsymbol{K} \cdot \boldsymbol{x} - wt + \theta) \tag{5}$$

obeying the transversality condition $\boldsymbol{K} \cdot \boldsymbol{E} = 0$. Clearly if the propagation vector $\boldsymbol{K}$ was parallel to the $x_1$-axis, a basic set of electric fields can be constructed with their polarizations pointing along any of the $x_2$, $x_3$, ..., $x_n$ Cartesian axes. It is this fact which was used in proposing recipe (3).

It is worth remarking that, in $n$-dimensions, the magnetic field $\boldsymbol{H}$ becomes a dyadic with $n(n-1)/2$ independent tensorial components read off from

$$H_{ij} = \partial A_j/\partial x_i - \partial A_i/\partial x_j \qquad i \neq j \tag{6}$$

whose physical interpretation is, however, more obscure than that of the electric field.

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## References

[1] Landsberg P T and De Vos A 1989 *J. Phys. A: Math. Gen.* **22** 1073–84